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Linearised $N = 2$ superfield supergravity

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Abstract. We present a formulation of linearised $N = 2$ supergravity in superfield form by solving linearised torsion constraints in superspace and identifying the irreducible representations generated by them with the multiplets which constitute the minimal $N = 2$ supergravity. After building the linearised Lagrangians for $N = 1$ supergravity (minimal and non-minimal) from the respective irreducible representations, we construct an unconstrained linearised superfield Lagrangian for the $N = 2$ case.

1. Introduction

One of the outstanding problems in extended supergravity is that of obtaining an extended superfield formulation of the cases with an internal SO(N) symmetry for $N \geq 2$. Such a formalism should allow the manipulation of both physical and auxiliary field variables, and might even aid in the discovery of the latter for $N \geq 3$. Furthermore, ultraviolet divergence cancellations discovered in low orders for $N \leq 8$ should become more transparent in an extended superfield language.

A complete geometrical superfield formulation of $N = 1$ supergravity has already been presented by various authors (Siegel and Gates 1979, Bedding *et al* 1979) and its quantum features analysed (Siegel 1979, Namazie and Storey 1979, Taylor 1979, Grisaru and Siegel 1981). The complete off-shell component structure for $N = 2$ has recently been given (de Wit *et al* 1980a, b) and a constrained superfield Lagrangian constructed (Sokatchev 1980). In order to develop a better understanding of the $N = 2$ case before proceeding to higher N, we will analyse here the construction of an extended superfield framework for $N = 2$ supergravity at the linearised level. This will involve describing appropriate techniques for classifying irreducible multiplets in a concise fashion, for writing down the corresponding Lagrangian, and for solving appropriate linearised torsion constraints (Castellani *et al* 1980, Stelle and West 1978, Wess 1979).

We start by developing our notation and applying it to the case $N = 1$. We especially consider the procedure of field redefinitions required to achieve the linearised Poincaré supergravity Lagrangian which selects certain irreps for both the minimal and non-minimal formulation. We then discuss the same features for $N = 2$, giving the field redefinitions producing the minimal auxiliary fields (de Wit and van Holten 1979) required for a linearised Poincaré Lagrangian to exist. In § 4 we solve linearised torsion constraints of algebraic form expected to be valid for higher N, and extend these to

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differential torsion constraints specialised to extract the $N=2$ Weyl and auxiliary multiplets. We then present a linearised superfield Lagrangian incorporating these irreps. Finally we make some remarks on the extension of the above work to higher N.

2. Superfield irreps for $N=1$ supergravity

In order to describe the strategy we propose to adopt for higher N, let us first consider the construction of a linearised superfield version of the simplest case of $N=1$ supergravity. To do that we use the fact that the known Poincaré spin content of an off-mass-shell massive irreducible representation (irrep) with superspin Y of the $N=1$ super-symmetry algebra is (Salam and Strathdee 1975) $(Y-\frac{1}{2},\ Y^2,\ Y+\frac{1}{2})$ (or $(0^2, \frac{1}{2})$ when $Y=0$). The irrep containing maximal spin 2 has therefore the value $Y=\frac{3}{2}$, with component fields A_a, $\psi_{a\alpha}$, h_{ab} where $\partial_a A_a = \partial_a \psi_{a\alpha} = \partial_a h_{ab} = h_{[ab]} = h_{aa} = (\gamma_a)_\alpha{}^\beta \psi_{a\beta} = 0$ and $\psi_{a\alpha}$ is Majorana. This Weyl multiplet can have a superfield representation on superfields Φ_j with external Poincaré spin j provided $j=2, \frac{3}{2}$ or 1. The value $j=1$ may be selected by one of various criteria. The most general arises from the fully nonlinear supergeometric approach to $N=1$ supergravity (Siegel and Gates 1979, Bedding *et al* 1979). The most specific, and more appropriate for our linearised approach to higher N superfield supergravities, makes use of torsion constraints which reduce the super-achtbein $E_A{}^M$ and super-connection $\Phi_{AB}{}^C$ to the covariant derivatives of a vector and lower external spin superfields.

On such a superfield we may easily extract the $Y=\frac{3}{2}$ irrep by means of the appropriate projection operator. The result is made most transparent by use of the basis functions of the $Y=0$ and $Y=\frac{1}{2}$ irreps on a scalar superfield. These are the eigenfunctions of $A=-(4\Box)^{-1}(\bar{D}D)^2$ with eigenvalues $A=1$ and 0 respectively, where D_α is the flat superspace covariant derivative of Salam and Strathdee and $\Box=\partial_a\partial_a$. Since $A^2=A$ we may construct the basis functions for $Y=0$ and $\frac{1}{2}$ as $A\pi D_\alpha$ and $(1-A)\pi D_\alpha$, where πD_α are the 16 possible independent products of D_α with itself. When $A=1$ the representations are still reducible and we split them with $G=-\bar{D}\partial\gamma_5 D$. Since $G^2=4P^4A$ the eigenvalues are $G=\pm 2P^2$ and the basis functions in this sector are $(1\mp G/2\Box)\pi D_\alpha$. By use of the identities for products of D_α's (Sokatchev 1975) we can show that these functions reduce to the sets

$$\begin{pmatrix} e_0 \\ u_\alpha \\ \omega_a \end{pmatrix} \equiv \begin{pmatrix} 2(1-A)\mathbb{1} \\ -4\Box^{-1}(1-A)(\mathrm{i}\partial D)_\alpha\,\mathbb{1} \\ \Box^{-1}(1-A)\bar{D}\mathrm{i}\gamma_a\gamma_5 D\,\mathbb{1} \end{pmatrix} \tag{1}$$

and

$$\begin{pmatrix} e_\pm \\ u_{\alpha\pm} \\ \omega_\pm \end{pmatrix} \equiv \begin{pmatrix} -\Box^{-1}\bar{D}D_\mp\bar{D}D_\pm\,\mathbb{1} \\ -2\Box^{-1}\bar{D}D_\mp D_{\alpha\pm}\,\mathbb{1} \\ \Box^{-1}\bar{D}D_\mp\,\mathbb{1} \end{pmatrix} \tag{2}$$

when $A=0$, 1 respectively. Our conventions and notation are $\eta_{ab}=\mathrm{diag}(1,-1,-1,-1)$, $\varepsilon_{0123}=-1$, $\gamma_5=\gamma_0\gamma_1\gamma_2\gamma_3$ and $\bar{D}D_\pm=\bar{D}_\pm D_\pm$, $D_\pm=\frac{1}{2}(1\pm\mathrm{i}\gamma_5)D$. On the RHS of (1) and (2) the θ derivatives act on the $\mathbb{1}$ (a constant in superspace), but not the space–time derivatives. Then the basis functions are differential operators in space–time and functions in θ space.

We may express the projection of a vector superfield V_a by means of the projectors π_0, π_1, onto the subspace with $A=0$ or 1 as, respectively,

$$\pi_0 V_a = e_0 B_a + \bar{u}^\alpha \chi_{\alpha a} + \omega_b A_{ba}, \tag{3}$$

$$\pi_1 V_a = e_\pm B_{\pm a} + \bar{u}^{\alpha\pm} \chi_{\alpha\pm a} + \omega_\pm A_\pm. \tag{4}$$

The $Y=\frac{3}{2}$ projection $V_a^{(3/2)}$ of V_a is obtained from the $A=0$ part of V_a by means of the projector $\frac{1}{3}(2-B)_{ab}$, where $B_{ab}=-(1/2\Box)\varepsilon_{abcd}\partial_c\bar{D}i\gamma_d\gamma_5 D$, its eigenvalues being 2 and -1. The product rules for D_α allow this to be reduced to (from now on we use momentum space instead of position space)

$$V_a^{(3/2)} = \tfrac{2}{3}P^{-2}e_0 A_a + 2^{-1/2}\bar{u}\not{P}^{-1}\psi_a + \omega_b[h_{ab} + (\mathrm{i}/3P^2)\varepsilon_{abcd}P_c A_d], \tag{5}$$

with the functions A_a, h_{ab} and $\psi_{a\alpha}$ satisfying the conditions noted earlier. We have included suitable factors of P_a in (5) so that A_a, h_{ab}, $\psi_{a\alpha}$ are local functions of V_a.

We may evaluate the Lagrangian density L for $V_a^{(3/2)}$,

$$L_{3/2} = \int \mathrm{d}^4\theta\, V_a^{(3/2)} P^2\, V_a^{(3/2)}, \tag{6}$$

by means of the identities

$$\begin{aligned} &\int \mathrm{d}^4\theta\, (e_0 A)(e_0 B) = -\tfrac{1}{2}AP^2 B, \\ &\int \mathrm{d}^4\theta\, (\omega_a A)(\omega_b B) = \tfrac{1}{2}A\bar{\eta}_{ab}B, \\ &\int \mathrm{d}^4\theta\, (u_\alpha A)(\mu_\beta B) = -A(\not{P}\eta)_{\alpha\beta}B, \end{aligned} \tag{7}$$

where A and B are commuting component fields, $\bar{\eta}_{ab}=\eta_{ab}-P_aP_b/P^2$, and we have dropped terms on the RHS which are total derivatives. The resulting super-Weyl Lagrangian density (6) is

$$L_{3/2} = -\tfrac{1}{3}A_a^2 + \tfrac{1}{2}h_{ab}P^2 h_{ab} + \tfrac{1}{2}\bar{\psi}_a\not{P}\psi_a. \tag{8}$$

In order to obtain the linearised super-Poincaré Lagrangian from the above it is necessary that we add to $L_{3/2}$ various scalars and spinors in order that the component fields A_{a1}, $\psi_{a\alpha}$ and h_{ab} are unconstrained, though have suitable gauge invariances. In particular, we need to add a scalar to A_a and h_{ab} and a Majorana spinor to $\psi_{a\alpha}$. These are contained minimally in a scalar superfield with $A=1$,

$$V_\pm = P^{-2}e_\pm A_\pm + \bar{u}^{\alpha\pm}\psi_{\alpha\pm} + 2\omega_\pm B_\pm. \tag{9}$$

We choose $V_- = V_+^*$ to reduce the number of components and have the Lagrangian density (using integration rules for $A=1$ basis functions similar to those in (7) for the $A=0$ sector)

$$L_0 = \int \mathrm{d}^4\theta\, V_- V_+ = A^* P^{-2} A + \bar{\psi}_+ \not{P}\psi_- + |B|^2, \tag{10}$$

where $A_+=A$, $B_+=B$. We write the constrained functions A_a, $\psi_{a\alpha}$, h_{ab} in terms of unconstrained ones V_a, $\phi_{a\alpha}$, f_{ab} as $A_a=\bar{\eta}_{ab}V_b$, $\psi_a=\bar{\eta}_{ab}\phi_b-\frac{1}{3}\bar{\eta}_{ab}\gamma_b\gamma_c\bar{\eta}_{cd}\phi_d$ and $h_{ab}=\bar{\eta}_{ac}\bar{\eta}_{bd}f_{cd}-\frac{1}{3}\,\bar{\eta}_{ab}\bar{\eta}_{cd}f_{cd}$. The associated relationships (to within divergences) for the

Lagrangian density $L_{3/2}$ are

$$A_a^2 = V_a^2 + P \cdot V P^{-2} P \cdot V, \tag{11a}$$

$$h_{ab} P^2 h_{ab} = 2L_{\mathrm{E}}(f) + 6P^2 b, \tag{11b}$$

$$\bar{\psi}_a \not{P} \psi_a = 2L_{\mathrm{RS}}(\phi) + \tfrac{2}{3}\bar{\lambda}\not{P}\lambda, \tag{11c}$$

where

$$\begin{aligned} L_{\mathrm{E}}(f) &= \tfrac{1}{2} f_{ab} P^2 f_{ab} - f_{ab} P_a P_c f_{cb} + f_{ab} P_a P_b f_{cc} - \tfrac{1}{2} f_{aa} P^2 f_{bb}, \\ b &= \tfrac{2}{3}^{1/2}(f_{aa} - P_a P_b / P^2 f_{ab}), \\ L_{\mathrm{RS}}(\phi) &= -\tfrac{1}{2}\varepsilon_{abcd}\bar{\phi}_a \gamma_5 \gamma_b P_c \phi_d, \\ \lambda &= \gamma \cdot \phi - \not{P}^{-1} P \cdot \phi. \end{aligned} \tag{12}$$

If we define

$$\operatorname{Re} A = \mathrm{i} P \cdot V, \qquad \operatorname{Im} A = \tfrac{3}{2}^{1/2} P^2 b, \qquad \psi = \sqrt{2}\lambda,$$

then

$$L_{3/2} - \tfrac{1}{3}L_0 = L_{\mathrm{E}}(f) + L_{\mathrm{RS}}(\phi) - \tfrac{1}{3}(V_a^2 + |B|^2). \tag{13}$$

We have therefore obtained the correct off-shell Lagrangian for Poincaré supergravity, with the associated minimal set of auxiliary fields B and V_a.

We note that we may regard (11) as defining a set of 'annihilation rules' for eliminating unwanted component fields from a superfield irrep. If we denote by $J_{\mathrm{p}}(J_A)$ a Bose field of Poincare spin J which has dimension $L^{-1}(L^{-2})$, we may rewrite $(11a)$ as the on-shell annihilation rule making the constrained vector A_a purely auxiliary,

$$1_A - 0_{\mathrm{p}} \sim 0, \tag{14}$$

where the 0 on the RHS of (14) denotes a field vanishing on-shell and the signs denote those in the kinetic energy terms. We will need to extend (14) when we turn to higher N, as we will see in § 3.

Our linearised superfield Lagrangian may be written in the compact form

$$L = \int \mathrm{d}^4\theta \; V_a P^2 (\pi_{ab}^{3/2} - \tfrac{1}{3}\pi_{ab}^{0}) V_b, \tag{15}$$

where V_a is now an unconstrained vector superfield $\pi_{ab}^{0} = -P_a P_b / P^2 \pi_0$, and the scalar superfield (9) is given by $|V_+|^2 = P \cdot V \pi^0 P \cdot V$.

We may choose a different irrep to annihilate A_a in (8). Instead of taking the $Y = 0$ irreps with Poincaré spin content $(0_A, \tfrac{1}{2}^{+\mathrm{i}}, 0_p)$, $(0_A, \tfrac{1}{2}^{-\mathrm{i}}, 0_{\mathrm{p}})$ we can try the $A = 0$, $Y = \tfrac{1}{2}$ irrep on a scalar superfield with content $(0_{\mathrm{p}}, \tfrac{1}{2}^{\pm\mathrm{i}}, 1_A)$. Two of these irreps are needed to contribute one scalar in $(11a)$ and another scalar in $(11b)$. In order to annihilate the two extra 1_A's and one $\tfrac{1}{2}^{\pm}$ we need two further $Y = 0$ irreps and the annihilations rule

$$\bar{\psi}\not{P}\psi - \bar{\lambda}\not{P}\lambda = \bar{\phi}\chi, \qquad \tfrac{1}{2} - \tfrac{1}{2} \sim 0, \tag{16}$$

where $\psi = \tfrac{1}{2}(\phi + \not{P}^{-1}\chi)$ and $\lambda = \tfrac{1}{2}(\phi - \not{P}^{-1}\chi)$. The net result is that on-shell $[(Y = \tfrac{3}{2}) + (Y = 0)^2 + (Y = \tfrac{1}{2})^2]$ again reduces to the correct supergravity spectrum. These irreps give the non-minimal set of auxiliary fields (Breitenlohner 1979). We may write the corresponding superfield Lagrangian as

$$L=\int \mathrm{d}^4\theta\,[V_a P^2(\pi^{3/2}_{ab}+\pi^0_{ab})V_b + A^*\pi^{1/2}A], \tag{17}$$

with component form

$$L=L_{\mathrm{E}}(f)+L_{\mathrm{RS}}(\phi)+U_a^2+V_a^2-A_a^2+\bar\lambda\psi+|B|^2, \tag{18}$$

after making field redefinitions similar to those in (11) and (16). The differences of signs of the auxiliary fields between (18) and (13) noted by Siegel and Gates (1979) are thus explained in terms of the different irreps used to achieve the correct on-shell $(2,\frac{3}{2})$ supergravity multiplet by means of the annihilation rules (14) and (16). This difference will be important in our consideration of $N=2$ supergravity for which the $N=1$ non-minimal set is appropriate.

3. Irreps for $N=2$ supergravity

The fundamental irrep with superspin $Y=0$ for $N=2$ may be obtained by the method of induced representations by applying the supersymmetry generators S_{1i}, S_{2i} to a basis state, $i=1,2$ being the SO(2) labels. The counting of states, denoted by (J^{P}, d), where J^{P} is the Poincaré spin and parity and d the SO(2) dimensionality, yields

$$(0_{\mathrm{P}}^+,1)+(\tfrac{1}{2}^{\pm\mathrm{i}},2)+(1_{\mathrm{P}}^-,1^{\mathrm{a}})+(0_{\mathrm{P}}^-,1)+(0_{\mathrm{A}}^-,\mathbf{2})+(0_{\mathrm{A}}^+,1),$$

where the 1 is the scalar representation of SO(2), 2 is the spinor representation, **2** is the traceless symmetric two-dimensional representation and 1^{a} is the antisymmetric one-dimensional representation. We distinguish between 1 and 1^{a}, even though no such distinction really exists in SO(2) because in the on-shell classification of states for supergravity the vector fields come in the antisymmetric second-rank tensor representation of SO(N). Whilst this representation reduces to the trivial one for $N=2$ it does not do so for $N>2$. We wish to develop our discussion of irreps and torsion constraints for $N=2$ in a manner most easily extendable to $N>2$, and thus make this distinction between 1 and 1^{a} throughout the paper.

The only irrep with maximum spin 2 has $Y=1$ (Taylor 1980, 1981) and the corresponding content is

$$(0_{\mathrm{P}}^+,1)+(\tfrac{1}{2}^{\pm\mathrm{i}},2)+(1_{\mathrm{P}}^+,1^{\mathrm{a}})+(1_{\mathrm{P}}^-,1^{\mathrm{a}})+(1_{\mathrm{A}}^+,1)+(1_{\mathrm{A}}^-,1^{\mathrm{a}})+(1_{\mathrm{A}}^+,\mathbf{2})+(\tfrac{3}{2}^{\pm\mathrm{i}},2)+(2_{\mathrm{P}}^+,1).$$

To obtain the correct on-shell assignment $(2_{\mathrm{P}}^+,1)+(\frac{3}{2}^{\pm\mathrm{i}},2)+(1_{\mathrm{P}}^-,1^{\mathrm{a}})$ from the $Y=1$ irrep we can use the annihilation rules of the previous section. Due to the relatively large number of spin-one fields to be removed we use the further annihilation rule

$$1_{\mathrm{P}}^+-1_{\mathrm{P}}^-\sim 0. \tag{19}$$

This may be proved by expressing an antisymmetric tensor A_{ab} as

$$A_{ab}=\mathrm{i}P_{[a}U_{b]}+\mathrm{i}\varepsilon_{abcd}P_cV_d,$$

and find that, to within total divergences,

$$A_{ab}{}^2=-(P_{[a}U_{b]})^2+(P_{[a}V_{b]})^2,$$

which proves (19). We can now use (11), (14), (16) and (19), and see that at least one

$Y=0$ irrep is needed to obtain $L_E(f)$ and $L_{RS}(\phi)$. However this does not remove the spin-$\frac{1}{2}$ from $Y=1$, so a further $Y=0$ irrep is needed. In order simultaneously to remove the remaining vectors we need to take this $Y=0$ irrep with the opposite dimension and with an external antisymmetric pair of SO(2) labels so that the content is

$$(0^+_A, 1^a)+(\tfrac{1}{2}^{\pm i}, 2)+(1^-_A, 1)+(0^-_A, 1^a)+(0^-_p, \mathbf{2})+(0^-_p, 1^a).$$

The resulting Lagrangian will have, in addition to the spin-2 and -$\frac{3}{2}$ contributions, the abelian field from the $Y=1$ multiplet together with exactly the auxiliary field structure of de Wit and Van Holten (1979).

Our above analysis has specified the superspin content of the required irreps as being $Y=0$ (twice) and $Y=1$. There are a number of such irreps, and since moreover we do not know the superfields on which the irreps are to be given, we turn to analysing torsion constraints at the linearised level to clarify this situation along lines similar to that for $N=1$.

4. Linearised torsion constraints

We start by linearising the geometrical objects in superspace, the vierbein $\hat{E}_M{}^A$, the connection $\hat{\Phi}_{MA}{}^B$, the torsion $\hat{T}_{AB}{}^C$ and the curvature, as follows:

$$\hat{E}_M{}^A = E_M{}^A + H_M{}^A, \qquad \hat{\Phi}_{MA}{}^B = \Phi_{MA}{}^B, \qquad \hat{T}_{AB}{}^C = \mathring{T}_{AB}{}^C + T_{AB}{}^C, \tag{20}$$

where $E_M{}^A$ and $H_M{}^A$ are the flat and linearised vierbein respectively, $\Phi_{MA}{}^B$ is the linearised connection and $\mathring{T}_{AB}{}^C$ and $T_{AB}{}^C$ the flat and linearised torsion. Our notation is $M=(m, \mu i)$, $i=1,\ldots,N$, a world index, where m and μi are the vector and fermionic components, and $A=(a, \alpha i)$ is the respective flat space index. We do not need to take curvatures into account since they can be expressed in terms of the other geometrical objects (Dragon 1979). The linearised torsion is (Wess 1979, Howe and Tucker 1979)

$$T_{BC}{}^A = D_{[B}H_{C\}}{}^A + \mathring{T}_{BC}{}^D H_D{}^A - H_B{}^D \mathring{T}_{DC}{}^A + (-1)^{bc} H_C{}^D \mathring{T}_{DB}{}^A + \Phi_{[BC\}}{}^A, \tag{21}$$

where $H_A{}^B = E_A{}^M H_M{}^B$, $\Phi_{AB}{}^C = E_A{}^M \Phi_{MB}{}^C$ and [} is a symmetrisation for a pair of spinor indices and an antisymmetrisation otherwise.

The structure group is chosen to be the Lorentz group. This means that we do not need to introduce a connection for the internal symmetry sector and thus it reduces the number of independent superfields. We also have supercoordinate invariance in superspace. Thus the linearised transformations are respectively

$$\delta_L H_B{}^A = L_B{}^A, \qquad \delta_L \Phi_{BC}{}^A = D_B L_C{}^A, \tag{22a}$$

$$\delta_G H_B{}^A = D_B \xi^A + \xi^C \mathring{T}_{CB}{}^A, \qquad \delta_G \Phi_{BC}{}^A = 0. \tag{22b}$$

We will also introduce local Weyl invariance later in order to select the auxiliary irreps, but we only require the torsion constraints to be invariant under them.

Since the connection appears without derivatives in (21) it can be solved algebraically by putting some components of the torsion equal to zero. There are several sets of equivalent torsion constraints of this type which differ by field redefinitions. Since we are going to use torsions or combinations of torsions which are independent of the connections, we do not need to specify this set of constraints.

We then impose the following set of connection-independent torsion constraints:

$$T_{\alpha+i\beta-i}{}^{a} = T_{\alpha+i\,a}{}^{a}$$
$$= C^{\beta-\beta'-}C_{\gamma'-\gamma-}T_{\alpha+i\beta'-j}{}^{\gamma'-j} - T_{\alpha+i\gamma-j}{}^{\beta-j} - N\mathrm{i}T_{\alpha+i\,a}{}^{b}(\sigma_b{}^{a})_{\gamma-}{}^{\beta-} = 0, \quad (23a)$$

$$T_{\alpha+i\beta+j}{}^{\gamma-k} = T_{\alpha+i\beta+j}{}^{a} = 0, \quad (23b)$$

$$\tilde{T}_{\alpha+i\beta-j}{}^{a} = 0, \quad (23c)$$

$$T_{\alpha+ij}{}^{j} = \tilde{T}_{\alpha+ij}{}^{k} = \bar{D}_{+[l}\sigma^{ab}T_{+i]jk} - \mathrm{cc} = \bar{D}_{+[i}T_{+i](jk)} + \mathrm{cc} = 0, \quad (23d)$$

$$\bar{D}_{+l}T_{+i[jk]} + \mathrm{cc} + (i \leftrightarrow l, \text{traceless}) = 0, \quad (23e)$$

$$\bar{D}_{+i}T_{+i[jk]} + \mathrm{cc} = 0, \quad (23f)$$

(plus a set of algebraic constraints to be solved for the connection) where $\sim$ means traceless on all the internal indices, $T_{\alpha+ij}{}^{k} = T_{\alpha+i\beta-j}{}^{\beta-k}$, cc stands for complex conjugation and $A_{ij} + (i \leftrightarrow j, \text{traceless}) = A_{(ij)} - \delta_{ij}A_{kk}$.

The constraints (23a) are algebraic (or conventional) and can be solved for general N. They express $H_a{}^{B}$ in terms of $H_{\alpha i}{}^{B}$

$$H_b{}^{a} = (1/2N)[\mathrm{i}(C\gamma_b)^{\alpha+\alpha-}D_{(\alpha+i}H_{\beta-i)}{}^{a} + (\gamma^a\gamma_b)_{\alpha+}{}^{\beta+}H_{\beta+i}{}^{\alpha+i} + (\gamma^a\gamma_b)_{\alpha-}{}^{\beta-}H_{\beta-i}{}^{\alpha-i}] \quad (24)$$

$$\mathrm{i}(\gamma^b C)_{\alpha+\beta-}H_{b\gamma-i}$$
$$= -[1/2(1+N)][D_{(\alpha+i}H_{\beta-j)\gamma-j} + D_{(\alpha+i}H_{\gamma-j)\beta-j}$$
$$+ \mathrm{i}N(\sigma_b{}^{a}C)_{\gamma-\beta-}D_{[\alpha+i}H_{a]}{}^{b}] + \tfrac{1}{2}C_{\beta-\gamma-}\{D_aH_{\alpha+i}{}^{a} - (1/2N)D_{\alpha+i}$$
$$\times[\mathrm{i}(C\gamma_a)^{\delta+\delta-}D_{(\delta+j}H_{\delta-j)}{}^{a} + 4\,H_{\delta+j}{}^{\delta+j} + 4H_{\delta-j}{}^{\delta-j}]\}. \quad (25)$$

The constraints (23b) can also be solved for general N and they introduce the prepotentials of the theory V^a and $\bar{V}^{\alpha}$

$$H_{\alpha+i}{}^{\beta-j} = D_{\alpha+i}\,\bar{V}^{\beta-j}, \quad (26)$$

$$H_{\alpha+j}{}^{a} = D_{\alpha+j}V^a + \mathrm{i}\,(\gamma^a C)_{\alpha+\beta-}\bar{V}^{\beta-j}. \quad (27)$$

This solution is invariant under ($S_{\alpha+\beta-} = (\gamma^a C)_{\alpha+\beta-}S_a$):

$$\delta V_a = S_a, \qquad \delta\bar{V}^{\alpha-j} = \tfrac{1}{4}\mathrm{i}D_{\beta+j}S^{\beta+\alpha-}, \quad (28)$$

if S_a satisfies

$$D_{\beta+j}S_{\alpha+\alpha-} + D_{\alpha+j}S_{\beta+\alpha-} = 0, \qquad D_{\alpha+j}D_{\beta+k}S^{\beta+\alpha-} = 0. \quad (29)$$

We also have supercoordinate invariance in superspace (22b) so that the full invariance of the theory is

$$\delta V^a = S^a + \xi^a \qquad \delta\bar{V}^{\alpha-j} = \tfrac{1}{4}\mathrm{i}D_{\beta+j}S^{\beta+\alpha-} + \bar{\xi}^{\alpha-j}. \quad (30)$$

We can now gauge $\bar{V}^{\alpha-j}$ completely away and also the real part of V^a, so that the solutions (26) and (27) are reduced to

$$H_{\beta+j}{}^{a} = -\mathrm{i}D_{\beta+j}V^a, \qquad (V_a)^* = V_a, \quad (31)$$

$$H_{\beta+j}{}^{\alpha-k} = 0, \quad (32)$$

which is still invariant under

$$\delta V_a = -\mathrm{Im}\,S_a. \quad (33)$$

The solution of these two sets of torsion constraints (23a), (23b) is a simple generalisation of the $N=1$ case (Howe and Tucker 1979).

At this point we note that while for $N=1$ V_a has maximum Poincaré spin-2 this is no longer true for $N>1$. In fact, the residual invariance (33) leaves just $Y=\frac{3}{2}$ on V_a for $N=1$, but it leaves irreps with $Y=\frac{3}{2}$, 2 for $N=2$ so that higher-spin fields are still present. We now show how a precise knowledge of the irrep content of V_a can be obtained. For $N=1$ the solution of (29) is

$$S_{\alpha+\beta-}=D_{\alpha+}\Lambda_{\beta-},$$

$\Lambda_{\beta-}$ being an unconstrained spinor superfield. It is easily shown, using the algebra of D's (Sokatchev 1975), that

$$(G+2P^2A)S_{\alpha+\beta-}=0,$$
$$(B_a{}^b+2A\delta_a{}^b-2\delta_a{}^b)S_b^{\perp}=0$$

where $S_a=S_a^{\perp}+S_a^{\|}$, $P^aS_a^{\perp}=0$, $(\delta_a{}^b-P_aP^b/P^2)S_b^{\|}=0$. Therefore the irrep content of S_a is

$$S_a^{\|}:\ Y=0,\ G=-2P^2; \qquad Y=\tfrac{1}{2},\ G=0$$
$$S_a^{\perp}:\ Y=\tfrac{1}{2},\ G=0; \qquad Y=1,\ G=-2P^2,$$

and since by (33) we can gauge from V_a the imaginary part of S_a, the only irrep left in V_a is $Y=\frac{3}{2}$, that is, the Weyl multiplet. A similar analysis for $N=2$ shows that the irrep content of S_a is

$$S_a^{\|}:\ Y=0,\ G=-4P^2,\ \tau=0; \qquad Y=\tfrac{1}{2},\ G=-2P^2,\ \tau^2=1$$
$$S_a^{\perp}:\ Y=\tfrac{1}{2},\ G=-2P^2,\ \tau^2=1; \qquad Y=1,\ G=-4P^2,\ \tau=0,$$

($\tau=1/P^2\bar{D}_1\mathrm{i}\not{P}D_2$ being the third Casimir operator of the $N=2$ algebra (Taylor 1980, 1981)) leaving in V_a a multitude of irreps so that V_a must be further constrained.

We still have to fix the Lorentz gauge in superspace (22a). It can be used to gauge away some components of $H_{\alpha+i}{}^{\beta+j}$, leaving

$$H_{\alpha+i}{}^{\beta+j}=\delta_{\alpha+}{}^{\beta+}(\delta_{ij}A+\tilde{H}_{ij})+(\sigma_{ab})_{\alpha+}{}^{\beta+}\tilde{H}_{ij}^{ab}. \tag{34}$$

The next constraint (23c) is mixed in the sense that it involves algebraic as well as differential constraints. We can rewrite it in the form $(C\gamma^b)^{\alpha+\beta-}\tilde{T}_{\alpha+i\beta-j}{}^a=0$ and take the irreducible pieces. The trace and symmetric part (on the vector indices) can be solved algebraically for $\tilde{H}_{ij}^{ab}$ and partially for $\tilde{H}_{ij}$. Then (34) can be written as

$$H_{\alpha+i}{}^{\beta+j}=\delta_{\alpha+}{}^{\beta+}(H_{ij}+\mathrm{i}\tilde{A}_{ij})+\tfrac{1}{16}\delta_{\alpha+}{}^{\beta+}\bar{D}_i\mathrm{i}\tilde{\gamma}^a\gamma_5D_jV_a-\tfrac{1}{8}\mathrm{i}(\sigma_{ab})_{\alpha+}{}^{\beta+}\bar{D}_i\mathrm{i}\tilde{\gamma}^a\gamma_5D_jV^b, \tag{35}$$

where $H_{ij}=-H_{ji}=H_{ij}^*$, $\tilde{A}_{ij}=\tilde{A}_{ji}=\tilde{A}_{ij}^*$. The antisymmetric part of the constraint gives a differential equation for V_a whose solution for $N=2$ is

$$V_a=\bar{D}_i\mathrm{i}\gamma_a\gamma_5D_jV, \tag{36}$$

where V is a pre-prepotential. An analysis of the irrep content of V_a shows that $V_a^{\|}$ can be gauged away and $V_a^{\perp}$ has $Y=1$, $G=\tau=0$ (singlet) which is the Weyl multiplet for $N=2$ (de Wit *et al* 1980a, b, de Wit and van Holten 1979).

5. Auxiliary irreps

Now we turn to the constraints which give rise to the auxiliary irreps. They can be best understood as partial gauge choices for the Weyl transformation in superspace (Gates *et al* 1980). We assume that only the torsion constraints are invariant under these transformations. The linearised form is

$$\delta H_{\alpha i}{}^{B} = \Lambda E_{\alpha i}{}^{B} + \tilde{L}_{ij} E_{\alpha j}{}^{B}, \tag{37}$$

where Λ is complex, and for $N = 2$, $\tilde{L}_{ij} = \mathrm{i}\tilde{M}_{ij} + \Lambda_{ij}$ with $\tilde{M}_{ij} = \tilde{M}_{ji} = \tilde{M}^{*}_{ij}$, $\Lambda_{ij} = -\Lambda_{ji} = \Lambda^{*}_{ij}$ The invariance of the algebraic constraints can be used to define the Weyl transformations on the connection and on the other components of the vierbein.

We can then write down the Weyl transformation for the components of the torsion which were not used till now, and count the irreps which are generated when the constraints are imposed. The situation is quite different from $N = 1$ (Gates *et al* 1980), since in that case there are only two possible partial gauge choices which give rise to the minimal and non-minimal set of auxiliary fields. For $N = 2$ there is a large range of partial gauge choices which do not seem to be equivalent among each other (regarding the irrep content), and for which we do not even know whether a solution for the corresponding constraints exist. In trying to obtain the minimal set of auxiliary fields (de Wit and van Holden 1979) we notice that a slight variation of the last constraint gives a non-minimal set.

The invariance of the constraints (23d) requires that

$$D_{\alpha+i}\Lambda = 0, \tag{38a}$$

$$\widetilde{\bar{D}_i D_j}(\Lambda^* + \Lambda) + \bar{D}_i \mathrm{i}\tilde{\gamma}_5 D_j(\Lambda^* - \Lambda) = 0, \tag{38b}$$

$$D_{\alpha+i}\tilde{M}_{jk} + D_{\alpha+j}\tilde{M}_{ki} + D_{\alpha+k}\tilde{M}_{ij} - \tfrac{1}{2}(\delta_{ij}D_{\alpha+l}\tilde{M}_{lk} + \delta_{ik}D_{\alpha+l}\tilde{M}_{lj} + \delta_{jk}D_{\alpha+l}\tilde{M}_{li}) = 0, \tag{38c}$$

$$\bar{D}_i\sigma^{ab}D_j\Lambda_{kl} = 0. \tag{38d}$$

Equations (38a) and (38b) tell us that Λ is chiral with $F = 4P^2$ ($F = \bar{D}_1 D_1 \bar{D}_2 D_2$ is an operator which reverses the chirality on chiral irreps and can be used to impose a reality condition on such irreps. Its eigenvalues are $F = \pm 4P^2$ (Taylor 1980, 1981)); equation (38c) requires that $\tilde{M}_{ij}$ has both $G = -4P^2$, and $-2P^2$, but since it is real the only solution is $\tilde{M}_{ij} = 0$. Finally equation (38d) implies that Λ_{ij} carries the irreps $Y = G = 0$, $\tau^2 = 4$, $\tau = 0$. The solution of the corresponding constraints is

$$A = \tfrac{1}{8}D_{\delta+j}D_{\delta-j}V^{\delta+\delta-} + T, \qquad D_{\alpha+}T = 0, \tag{39a}$$

$$T = \tfrac{1}{4}\bar{D}_{+i}D_{+i}\bar{D}_{+j}D_{+j}V + S, \qquad D_{\alpha+}S = \widetilde{\bar{D}_i D_j}S = 0, \tag{39b}$$

$$\tilde{A}_{ij} = 0, \tag{39c}$$

$$\bar{D}_i\sigma^{ab}D_j H_{kl} = 0. \tag{39d}$$

In (39a) T is a compensating chiral superfield which is further constrained with solution given by (39b) (unlike $N = 1$ when it is chiral but otherwise unconstrained). Then the solution (39b) involves the true compensating superfield for $N = 2$. The conditions on S show that it carries the irrep $Y = \tau = 0$, $F = 4P^2$, that is, the vector-gauge multiplet (de Wit and van Holten 1979). Equation (39d) implies that H_{ij} carries the irreps $Y = G = 0$, $\tau^2 = 4$ and $\tau = 0$. At this stage we have fixed all Weyl parameters and we have a non-minimal formulation with $56 + 56$ fields.

The last two constraints (23*e*) and (23*f*) impose further conditions on the Weyl parameter Λ_{ij}:

$$\widetilde{\bar{D}_i D_j} \Lambda_{kl} = 0, \tag{40a}$$

$$\bar{D}_i D_i \Lambda_{kl} = 0. \tag{40b}$$

Equation (40*a*) removes $Y = G = 0$, $\tau^2 = 4$, while (40*b*) removes $Y = 1$, $G = \tau = 0$ (the tensor-gauge multiplet (de Wit and van Holden)) giving rise to the minimal and a non-minimal set of auxiliary fields with 40+40 and 48+48 fields respectively. The corresponding constraints on H_{ij} are then

$$\widetilde{\bar{D}_i D_j} H_{kl} = 0, \tag{41a}$$

$$\bar{D}_i D_i H_{kl} = 0. \tag{41b}$$

These formulations depend on a real scalar superfield V, a scalar superfield S (constrained by (39*b*)), both being scalars under SO(2) and a real scalar superfield H_{ij} in the antisymmetric representation of SO(2) (constrained by (39*d*), (41*a*) or (41*b*)). Unlike the non-minimal $N = 1$ formulation in superspace (Siegel and Gates 1979, Bedding *et al* 1979, Gates 1981). we do not have a compensating spinor superfield. Also the structure of the constraints is similar to the minimal $N = 1$ case in the sense that upon reduction we obtain the constraints for the minimal set. In fact the reduction does not distinguish between the minimal and non-minimal $N = 2$ cases.

6. $N = 2$ linearised superfield Lagrangians

Our analysis of torsion constraints of §§ 4 and 5 has somewhat narrowed down the choice of irreps to three possible choices, all containing the minimal set described in § 3. We showed that the $Y = 1$ Weyl irrep containing the graviton and gravitino is in the scalar superfield V and that there are auxiliary $Y = 0$ irreps in S and H_{ij}. The irrep in S has $F = 4P^2$, whilst the irreps in H_{ij} distinguish the three possibilities from each other. In both the 48+48 or 56+56 scheme H_{ij} has the $\tau^2 = 4$, $Y = 0$ irrep as well as the $Y = G = \tau = 0$ irrep in the latter case.

The spin-SO(2) content of the $Y = 0$, $\tau^2 = 4$ irrep is obtained from the $Y = G = \tau = 0$ irrep by direct product of the latter with the two-dimensional symmetric traceless representation of SO(2), to give for the former

$$(0^-_{\mathrm{A}}, \mathbf{2}) + (0^+_{\mathrm{A}}, \mathbf{2}) + (0^-_{\mathrm{P}}, \mathbf{2}) + (0^+_{\mathrm{P}}, \mathbf{2}) + (0^-_{\mathrm{P}}, 1) + (0^-_{\mathrm{P}}, 1^{\mathrm{a}}) + (1^-_{\mathrm{A}}, \mathbf{2}) + (\tfrac{1}{2}^{\pm \mathrm{i}}, 2) + (\tfrac{1}{2}^{\pm \mathrm{i}}, 2).$$

The case of 48+48 can easily be disposed of by the observation that there is a single unwanted spin-$\frac{1}{2}$ SO(2)-doublet which remains on-shell. The case of 56+56 is more subtle, but the crucial difficulty arises from the pair of physical SO(2)-doublet scalars in the $\tau^2 = 4$ irrep. There is only one SO(2)-doublet vector, and that is in the Weyl multiplet, so at least one of those scalars will persist on-shell. We are left therefore solely with the minimal 40+40 scheme.

We can now rapidly write down the linearised superfield Lagrangian which corresponds to this situation. It is necessary to take the projectors (Taylor 1980, 1981) on to the appropriate irreps in V, S and H_{ij} (which solves the remaining torsion constraints),

$$\pi_1 V = \pi_{Y=1} V = [\tfrac{3}{4}(1-A_1)(1-A_2) - (1/4P^4)J]V,$$
$$\pi_2 S = \pi_{F=4P^2} S = \{(R/16P^4) + \tfrac{1}{4}A_1A_2 + (1/16P^2)[\bar{D}_1D_1\bar{D}_2D_2 - \bar{D}_1\mathrm{i}\gamma_5D_1\bar{D}_2\mathrm{i}\gamma_5D_2]\}S, \tag{42}$$
$$\pi_3 H_{ij} = \pi_{Y=G=\tau=0} H_{ij} = (1/16P^4)(-R + P^2W + 4P^4A_1A_2)H_{ij},$$

where

$$\begin{aligned} A_i &= (1/4P^2)(\bar{D}_iD_i)^2, \\ J &= \tfrac{1}{4}(P^2\bar{D}_1\mathrm{i}\gamma_a\gamma_5D_1\bar{D}_2\mathrm{i}\gamma_a\gamma_5D_2 - \bar{D}_1\mathrm{i}\not{P}\gamma_5D_1\bar{D}_2\mathrm{i}\not{P}\gamma_5D_2), \\ R &= \bar{D}_1\mathrm{i}\not{P}\gamma_5D_1\bar{D}_2\mathrm{i}\not{P}\gamma_5D_2, \\ W &= \bar{D}_1D_1\bar{D}_2D_2 - \bar{D}_1\mathrm{i}\gamma_5D_1\bar{D}_2\mathrm{i}\gamma_5D_2. \end{aligned} \tag{43}$$

The superfield Lagrangian is thus finally

$$L = \int \mathrm{d}^4\theta\, (V\pi_1V - S\pi_2S - H_{ij}\pi_3H_{ij}). \tag{44}$$

We may write down the component expansions in basis functions for the corresponding superfields. For $\pi_1 V$ we first expand V in products of basis functions e_0, u_α, ω_a in each of θ_1 and θ_2 respectively, and then project onto the $J = P^4$ subspace by means of the projector $(1/4P^4)(3P^4 + J)$. After a little algebra the resultant superfield is

$$\begin{aligned} \pi_1 V = {} & [(P^2)^{-1}e_0e_0 - \tfrac{1}{3}\omega_a\omega_a]A + (P^2)^{-1}(e_0\omega_a + \omega_ae_0)A_a \\ & + b_{aij}B_{aij} + (2P^2)^{-1}\bar{u}_i\mathrm{i}\sigma_{ab}u_jT_{abij} \\ & + (2P^2)^{-1}\bar{u}_i\mathrm{i}\gamma_au_jA_{aij} + (\omega_{(a}\omega_{b)} - \tfrac{2}{3}\bar{\eta}_{ab}\omega_c\omega_c)h_{ab} \\ & + [\omega_a(\bar{u}_i\not{P}^{-1})^\alpha + e_0(\bar{u}_i\sigma_{ab}P_b\gamma_5\not{P})^\alpha]\psi_{a\alpha i}, \\ b_{aij} = {} & (P^2)^{-1}\begin{pmatrix} e_0\omega_a - \omega_ae_0 + \mathrm{i}\varepsilon_{abcd}P_b\omega_c\omega_d & b_{a21} \\ -\tfrac{1}{2}\bar{u}\mathrm{i}\gamma_a\gamma_5u & -b_{a11} \end{pmatrix}, \end{aligned} \tag{45}$$

with $P^aA_a = P^aB_{aij} = P^aA_{aij} = P^ah_{ab} = P^a\psi_a = h_{[ab]} = 0$. For $\pi_2 S$ we may combine the $G = +4P^2$ irrep constructed from products of $(e_+, u_{\alpha+}, \omega_+)$ basis functions and its complex conjugate and project out the $F = +4P^2$ part, giving

$$\begin{aligned} \pi_2 S = {} & [(2P^2)^{-1}e_+e_+ + 2\omega_+\omega_+ + \mathrm{cc}]A \\ & + \mathrm{i}[(2P^2)^{-1}e_+e_+ - 2\omega_+\omega_+ + \mathrm{cc}]B + (2P^2)^{-1}(e_+\omega_+ + \omega_+e_+ + \mathrm{cc})F \\ & + b_{ij}B_{ij} + (4P^2)^{-1}\bar{u}_i\mathrm{i}\sigma_{ab}u_jF(A)_{abij} \\ & + (P^2)^{-1}\{[(e_+\bar{u}^{\alpha+i} - 2\omega_-(\bar{u}_-\mathrm{i}\not{P})^{\alpha+}]\psi_{\alpha+i} + \mathrm{cc}\}, \\ b_{ij} = {} & \mathrm{i}\begin{pmatrix} e_+\omega_+ - \omega_+e_+ - \mathrm{cc} & b_{21} \\ -\tfrac{1}{2}\bar{u}\mathrm{i}\gamma_5u & -b_{11} \end{pmatrix}. \end{aligned} \tag{46}$$

Finally $\pi_3 H_{ij}$ is constructed by projecting the $G = 0$, $A = 2$ superfield constructed from products of $(e_+, u_{\alpha+}, \omega_+)$ and $(e_-, u_{\alpha-}, \omega_-)$ basis functions onto the subspace with

$W = 8P^2$ to give

$$\begin{aligned}\pi_3 H_{ij} = {} & (P^2)^{-1}(e_+\omega_- + \omega_+ e_- + \mathrm{cc})A_{ij} + \mathrm{i}(P^2)^{-1}(e_+\omega_- - \omega_+ e_- - \mathrm{cc})F_{ij} \\ & + [(2P^2)^{-1}e_+e_- + 2\omega_+\omega_- + \mathrm{cc}]D_{ij} + d_{k[i}B_{j]k} + (2P^2)^{-1}\bar{u}_i \mathrm{i}\gamma_a u_j V_a \\ & + (2P^2)^{-1}[(e_-\bar{u}^{\alpha+[i} - 2\omega_+(\bar{u}_{-[i}\not{P})^{\alpha+})\psi_{\alpha+j]} + \mathrm{cc}], \qquad P^a V_a = 0, \qquad (47) \\ d_{ij} = {} & \mathrm{i}(4P^2)^{-1}\begin{pmatrix} e_+e_- + 4P^2\omega_-\omega_+ - \mathrm{cc} & d_{12} \\ \bar{u}\mathrm{i}\not{P}\gamma_5 u & -d_{11}\end{pmatrix}.\end{aligned}$$

The field redefinitions are accomplished exactly as described in § 3, where the first and second $Y = 0$ multiplets are $\pi_2 S$ and $\pi_3 H_{ij}$ as expressed above.

The superfield Lagrangian (44) may be written, after recombination, as

$$\begin{aligned}&[(2_\mathrm{p}^+, 1) - (0_\mathrm{p}^+, 1)] + [(\tfrac{3}{2}^{\pm\mathrm{i}}, 2) - (\tfrac{1}{2}^{\pm\mathrm{i}}, 2)] + (1_\mathrm{p}^-, 1^\mathrm{a}) \\ &\quad - [(1_A^+, \mathbf{2}) - (0_\mathrm{p}^-, \mathbf{2})] - [(1_A^+, 1^\mathrm{a}) + (0_\mathrm{p}^-, 1^\mathrm{a})] - [(1_A^+, 1) - (0_\mathrm{p}^-, 1)] \\ &\quad + [(1_A^-, 1) - (0_\mathrm{p}^-, 1)] - (0_A^-, \mathbf{2}) - (0_A^+, 1) - (0_A^-, 1^\mathrm{a}) - (0_A^+, 1^\mathrm{a}) \\ &\quad - [(1_\mathrm{p}^+, 1^\mathrm{a}) - (1_\mathrm{p}^-, 1^\mathrm{a})] + [(\tfrac{1}{2}^{\pm\mathrm{i}}, 2) - (\tfrac{1}{2}^{\pm\mathrm{i}}, 2)].\end{aligned}$$

According to our discussion of §§ 2 and 3, the first three terms are the linearised Einstein, Rarita–Schwinger and Maxwell Lagrangian, whilst all the remainder are purely auxiliary, giving the auxiliary fields of de Wit and van Holten (1979).

7. Concluding remarks

Our superspace analysis is not directly comparable with other formulations (Castellani *et al* 1980, Stelle and West 1978, Wess 1979) which are based on an SU(2) group (either gauged in superspace or not), since the defining conditions for the irreps depend strongly on which group is being used. This explains why some of our differential torsion constraints differ from those presented in Stelle and West (1978) although the irrep content is the same.

We also meet a different situation from $N = 1$ where the breaking of the super-Weyl invariance leads us to non-minimal formulations for which there do not exist Poincaré Lagrangians. However, because of the great number of super-Weyl partial gauge choices, we cannot conclude that a non-minimal formulation does not exist.

We have tried to divide our torsion constraints into those expected to be applicable to higher N and those special to $N = 2$. We may attempt to extend the latter, as well as the former, to higher N. However, more recent work (Rivelles and Taylor 1981) indicates the need for the presence of central charges in order to go off-shell for $N \geqslant 3$ supergravities. Because of this, considerable changes of constraints may well be necessary in order to go beyond $N = 2$.

We must recognise that our recent results on the need for spin-reducing central charges for higher N (Rivelles and Taylor 1981) indicate that both classical and quantal features of the $N = 2$ case will be modified considerably even at $N = 3$. We propose therefore to investigate this and higher-N cases more fully by the techniques we have used successfully for $N = 2$.

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